Unemployment and Workplace Safety in a Search and Matching Model*

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Abstract

This paper develops a model with search to examine the determinants of the amount of capital purchased by a firm for workplace safety and investigates a relationship between unemployment and the incidence of work-related injury or illness. Productivity improvement encourages firms' entry and therefore lowers the unemployment rate but instead raises the employment rate, including the fraction of absent workers. On the other hand, productivity improvement encourages firms to buy more capital for workplace safety. This biases the distribution of employed workers toward non-injured workers, lowering the fraction of absent workers. These two effects determines the relationship between unemployment and the incidence of work-related injuries or illnesses.

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1 Introduction

This paper develops a model with search to consider firms' determinants of the amount of capital for workplace safety and to explore a relationship between unemployment and the incidence of work-related injury or illness. Assuming that the probability of a worker being injured or taken ill at work sites depends negatively on the amount of capital for workplace safety, there is trade-off for firms between the cost of its capital and the risk of losing a worker.

According to Robinson (1988) and Poteet and Didonato (2001), the number of work-related injuries is positively associated with employment size. They argued that during an economic boom in which labor demand exceeds labor supply, firms hire even inexperienced workers who are more likely to be injured at the work sites, and therefore that both the employment size and the flow of absent employed workers because of work-related injuries increase. In other word, the unemployment rate and the flow rate of absent employed workers are *negatively* correlated, controlling for the labor force.

However, Ussif (2004) undertook an international comparative study using time-series data between 1970 and 1999 from several countries and found an opposite relationship; that is, as employment size increased, the number of work-related injuries decreased. In other words, the unemployment rate and the flow rate of absent employed workers are *positively* correlated, controlling for the labor force. Additionally, he found movements of these two rates in the same direction if a time trend was controlled, which turns to be consistent with the evidence from Robinson (1988) and Poteet and Didonato (2001). Ussif (2004) concluded that the number of work-related injuries had declined because of the technical advancement of workplace devices and environments captured by the time trend.

Using *LABORSTA* (ILO) data from the US, Germany and Japan, the first graphs of Figure 1-3 present movements of relative deviations of the unemployment rate and the fraction of work-related injured workers from their corresponding trends. For the US and Germany, the relative deviations moved in the opposite direction over the sample period, implying a negative correlation between the unemployment rate and the fraction of work-related injured workers. The same pattern is seen after 1995 in Japan, but the deviations moved in the same direction before 1995. It means a positive correlation between the unemployment rate and the fraction of work-related injured workers.

These results showed that the number of work-related injuries is not determined only by the scale effect of employment size. There are many other factors affecting the rate of work-related injury, including employer practice at the work site, employee training, the role of unions, the technical advancement of work goods and environments as pointed out by Ussif (2004), the provision of mandates for safety, and investment in safety. This paper focuses attention on the last element among them, that is, investment in safety and develops a model that endogenizes the probability for a worker being subject to a work-related injury or illness in which a firm decides how much capital to purchase for workplace safety. Our contribution is to provide a new dimension to explain the relationship between the unemployment rate and the incidence of work-related injury or illness by incorporating the determinants of capital for workplace safety into a search-matching model. This paper does not discuss the role of mandates to keep workplaces safe by, for example, the Occupational Safety and Health Administration (OSHA) in the United States and its effect on labor market conditions.[1]

There are a few theoretical studies in this field. Holmlund (2005) presented a model with individual search behavior and a decision on sickness absence with the framework of the stochastic utility function of sickness and analyzed the impact of social insurance on a worker's labor supply decision. He focused on an individual worker's decision on labor supply and sickness absence. Our paper instead focuses on the determinant of the amount of capital purchased by *firms* for workplace safety to reduce the risk of losing employed workers because of work-related injuries or illnesses. One of the main findings in Holmlund (2005) is that an increase in sickness benefits raises the value of participating in the labor force, thereby resulting in an increase in employment size. Engström and Holmlund (2007) extended to a general equilibrium model with search by incorporating absence

[1] Jolls (2008) surveyed both theoretical and empirical studies on the effects of OSHA and compensation programs of work-related injuries.

from work as an additional state.[2] They derived the optimal compensation package to maximize the expected profit affected by the number of job applications and sick workers' determinants of absence from work under the condition that accidents randomly arrived at workers. They provided the welfare analysis and compared with alternative social insurance policies.

Empirical studies in this area have thus far explored the effect of OSHA on work-related injuries using state-level, industry-level or plant-level data from the US.[3] Overall, the effect of OSHA enforcement on the rate of work-related injuries was modest in the US (Viscusi 1979 1986, Bartel and Thomas 1985). In contrast, Scholz and Gray (1990) found a significant relationship between OSHA enforcement and the rate of work-related injuries using plant-level data of firms that were frequently inspected. According to the recent study by Mendeloff (2005), its significant relationship was observed in the early 1990s but it disappeared afterward.

Our findings are summarized below. Productivity improvement encourages firms to enter the labor market, which makes more competitive for firms to hire a worker. Firms that buy capital for workplace safety *before* meeting unemployed workers are then discouraged from doing so. On the other hand, productivity improvement leads to an increase in profit. In an environment in which firms have to recompense for a loss from a worker's absence, an increase in profit implies an increase in the opportunity cost of work-related injury or illness, which induces firms to purchase more capital for workplace safety to reduce the risk of work-related injury or illness. Which effect is more dominant is theoretically ambiguous. If the capital elasticity of the injury rate is sufficiently inelastic, the letter effect is more dominant over the former effect, in case of which productivity improvement raises the amount of capital for workplace safety and then lowers the fraction of absent employed workers. This is referred to as the *capital effect*. The amount of capital for workplace safety determines the distribution of employees between active employed workers and absent employed workers, but does not affect the unemployment

[2] Barmby et al. (1994) presented a model in which the wage is endogenously determined within an efficiency wage setting. They show that the wage is adjusted to affect the decision on sickness absence. Garibaldi and Wasmer (2001) also built a multistage model where the wage is endogenously determined.

[3] Smith (1992) surveyed empirical studies in the early 1990s.

rate.

There is another effect of productivity improvement through the labor market conditions. Productivity improvement encourages firms' entry, which raises labor market tightness and thereby lowers the unemployment rate. The larger employment size raises the fraction of workers who are absent from work because of work-related injuries or illnesses, holding the transition rate of being injured or taken ill fixed. This is referred to as the *scale effect*.

Overall, productivity improvement unambiguously lowers the unemployment rate but provides an ambiguous effect on the fraction of absent employed workers. If the scale effect is dominant over the capital effect, the unemployment rate and the fraction of absent employed workers move in opposite directions in response to productivity improvement. However, in a case in which the capital effect is dominant, the same decreasing trends in the unemployment rate and the fraction of absent employed workers are observed as productivity improves.

The same argument can be applied to the flow of absent employed workers. In a case of which productivity improvement leads to an increase in the amount of capital for workplace safety, the transition rate from active employment to absence is lower, thus resulting in the lower flow rate of absent employed workers, holding the employment rate fixed (capital effect). On the other hand, the larger employment rate led to by firms' entry raises the flow rate of absence employed workers (scale effect) holding the probability of being injured or taken ill fixed. If the scale effect is dominant over the capital effect, the unemployment rate and the flow rate of absent employed workers are negatively correlated as argued by Robinson (1988) and Poteet and Didonato (2001). In the reverse case, these two rates are positively correlated, which is consistent with Ussif (2004).

The amount of capital for workplace safety purchased by firms is a key variable that determines the relationship between unemployment and the incidence of work-related injury or illness. The puzzling empirical results could be partially explained by endogenizing the determinant of capital for workplace safety.

The model is extended to analyzing effects of various policy parameters (replacement rate, disability insurance payment and premium, and unemployment benefit). Among them, the effect of unemployment benefits is particularly noteworthy. To the best of our knowledge, unemployment benefit and workplace safety have been thus far discussed separately. We here show the strong linkage in policy between them. An increase in the unemployment benefit raises a worker's reservation wage and thereby the wage because the state of unemployment is more attractive for workers. It also raises the cost that firms would have incurred if their workers had been injured or taken ill at the work site because its cost is positively proportional to the wage. Therefore, firms are encouraged to increase capital for workplace safety to reduce the likelihood of injury or illness at the work site. However, there is the opposite view; that is, the increasing wage lowers the profit gained by firms, which discourages them from purchasing capital for workplace safety. Additionally, a decrease in the profit encourages firms to exit. Less competitiveness provides firms with incentives to raise capital for workplace safety because the probability of meeting a worker is higher. If the positive effects are dominant over the negative one, we recognize the novel result that an increase in unemployment benefit raises the amount of capital for workplace safety and thereby lowers the likelihood that employees are injured or taken ill at the work site.

We compare the effects on the flow rate of employees who become absent from work because of work-related injuries through changes in capital for workplace safety (capital effect) and labor market tightness (scale effect), using time-series data from the US, Germany and Japan.

The next section presents a matching model with an endogenous determinant of capital for workplace safety. Section 3 illustrates comparative statics exercises and discusses a determinant of a relationship between unemployment and the incidence of work-related injury or illness by productivity improvement. The effects of various social insurance programs are analyzed in Section 4. The estimated results are presented in Section 5 to confirm implications obtained from the model. The final section provides concluding remarks.

2 The Model

We consider a continuous-time model with matching in which there are a continuum of risk-neutral workers and a continuum of risk-neutral firms. The measure of workers is normalized to one. Workers are infinitely lived and homogeneous with respect to preferences to work. At any moment, a worker is either unemployed, employed, or absent from work because of work-related injury or illness. An employed worker is injured or taken ill at the work site and absent from work at a Poisson rate $\lambda(k)$, where k represents capital, with its price normalized to one, purchased by a firm to improve conditions of workplace safety. Work-related injuries and illnesses are defined here by immediate health hazard in the course of work that forces workers to be absent from work for treatment such as lower-back pain, cuts, bruises, broken bone, falls, struck by objects, mental illness and so on, but not by long-term latent health hazard from work such as pneumoconiosis. Following Acemoglu and Shimer (1999), a firm that creates job vacancies through free entry decides on how much capital k to buy for each job vacancy *before* meeting an unemployed worker.[4] A construction company installs handrails and safety net for falls at a construction site before hiring workers. A manufacturing company complies an instruction manual for workplace safety and health and purchases safety machines and devices when constructing a factory. We assume that λ is characterized by $\lambda'(\cdot) < 0$, $\lambda''(\cdot) > 0$, $\lambda(0) = \overline{\lambda} \leq \infty$ and $\lim_{k \to \infty} \lambda(k) = \underline{\lambda} \geq 0$. As a firm buys capital to improve conditions of workplace safety, the probability of an employed worker being injured or taken ill is reduced at a decreasing rate.

There is search-matching friction. The unemployed and job vacancies are matched randomly according to a matching function, $m(u, v)$ where u is the number of the unemployed and v is the measure of job vacancies across all firms. The matching function is assumed to exhibit constant returns to scale, implying that the rate at which a vacancy encounters an unemployed worker is computed by $m(u, v)/v = m(u/v, 1) \equiv q(\theta)$ where $\theta \equiv v/u$ is labor market tightness, while the rate at which an unemployed worker matches with a job vacancy is represented by $\theta q(\theta)$. Note that $q(\theta)$ is decreasing in θ;

[4] Acemoglu and Shimer (1999) determined how much capital to buy for production in a search model.

that is, $q'(\theta) < 0$.

A job is destroyed at an exogenous Poisson rate δ, and then a worker becomes unemployed and begins to search for a job. Note here that a firm loses even a vacancy when the job is destroyed, which is different from the standard model with matching. Both workers and firms discount the future at the common rate r.

Various value functions are developed below. We begin with the value for an employed worker of engaging actively in work as follows:

$$rW(w) = w + \lambda(k)[W_a(w) - W(w)] + \delta[U - W(w)]. \tag{1}$$

The instantaneous utility is linear with earnings. The second term on the right-hand side of equation (1) represents the expected capital loss incurred by being injured or taken ill, and the third term indicates the expected capital loss of being unemployed. In a similar manner, the value for an employed worker of being absent from work because of work-related injury or illness is defined by:

$$rW_a(w) = w + \alpha[W(w) - W_a(w)] + \delta[U - W_a(w)]. \tag{2}$$

For convenience, a worker and firm have a contract that if being absent from work due to injury at the work site, the worker is compensated in full for her earning w. The absent employed worker heals and returns to work at an exogenous Poisson rate α.[5] Note that disutility incurred by the absent employed worker is ruled out in this model without loss of generality. Equations (1) and (2) thus show that the employed worker is indifferent between working and absence from work due to work-related injury or illness because the absent worker is fully compensated. Therefore, these values can be expressed in a standard form:

[5] In fact, the rate of return to work is not exogenous. It depends largely on the amount of compensations as well as the extent of injury or illness if it is difficult to observe whether or not absent workers heal from injury or get well from sickness. Meyer, Viscusi, and Durbin (1995) undertook a natural experiment and found that an increase in compensations received by absent employed workers extends the duration of absence. Similar results are obtained in Ehrenberg (1988) and Krueger (1990).

$$rW(w) = rW_a(w) = w + \delta[U - W(w)].$$

A worker's surplus is then calculated by:

$$W(w) - U = \frac{w - rU}{r + \delta}. \quad (3)$$

The value of unemployment is given as usual by:

$$rU = \theta q(\theta)[W(w) - U]. \quad (4)$$

At any moment, an unemployed worker, who is assumed to receive no instantaneous utility, meets a firm with a vacant job at the transition rate $\theta q(\theta)$.

Next we discuss value functions for a firm. We consider a firm to be a collection of individual jobs. At any time in point, jobs are either occupied, unfilled, or inactive because employed workers are absent due to work-related injuries or illnesses. We assume that firms operate under constant-returns-to-scale production technology with respect to labor input. This assumption assures that jobs are independent of one another.

The value of a job being occupied and active can be expressed as:

$$rJ(w,k) = p - w + \lambda(k)[J_a(w,k) - J(w,k)] - \delta J(w,k). \quad (5)$$

A matched pair produces p instantaneously. The second term on the right-hand side of equation (5) represents the expected capital loss of a job being inactive because a worker is absent from work owing to work-related injury or illness. The third term indicates the expected capital loss of a job being destroyed.

Similarly, the value of an occupied job being inactive because of work-related injury or illness is:

$$rJ_a(w,k) = -w + \alpha[J(w,k) - J_a(w,k)] - \delta J_a(w,k). \quad (6)$$

We assume that there are no disability insurance programs, and therefore that the firm has to compensate the absent worker for her/his wage according to the contract. It implies that renegotiation over the wage is not allowed between a worker and a firm after the injury or illness shock. We believe that this assumption is acceptable. In reality, many firms join the federal or state disability insurance program with compulsory payroll deductions. If own employees are injured or taken ill at the work site, they are compensated through its program.[6] Because the disability insurance program is mainly financed by firms, it is interpreted that firms indirectly bear the burden of compensation. Even though own employees become absent from work because of injury or illness that occurs out of the work site, many firms provide with absence leave programs with payments. According to the survey conducted by Japanese Ministry of Health, Labour and Welfare (MHLW) in January 2008, 58.6% of surveyed Japanese firms have own absence leave programs, and 41.1% of them keep paying average 85.8-93.6% of salaries to absent employees. Putting down to surveyed firms over 1,000 employees, 85.3% have own absence leave programs, and 56.8% pays 88.5-91.8% of salaries to absent workers.[7]

The job turns out to be active at the rate α. Equations (5) and (6) give the following equation:

$$J(w,k) = \frac{(r+\alpha+\delta)p - (r+\alpha+\delta+\lambda(k))w}{(r+\delta)(r+\alpha+\delta+\lambda(k))}. \tag{7}$$

The value of a vacancy is given by:

$$rV(k) = q(\theta)[J(w,k) - V(k)] - \delta V(k). \tag{8}$$

A vacancy is filled at the transition rate $q(\theta)$, but destroyed at the rate δ, similar to an occupied job.

The free entry condition ensures that the value of a vacancy equals k in equilibrium ($V(k) = k$). Using equation (8), the labor demand equation is thus given by:

[6] We consider these effects on capital for workplace safety and labor market tightness later

[7] See *Rodo Sinbun* (Labour Newspaper) No. 2688 (July 14, 2008) in Japanese published by *Rodo Sinbunsha.*

$$k = \frac{q(\theta)J(w,k)}{r + \delta + q(\theta)}. \tag{9}$$

A firm and a worker consummate a match if and only if the joint surplus gained through this match is nonnegative, and then they share the joint surplus according to the Nash bargaining rule. Assuming that the worker's share of the surplus is defined by $\beta \in [0,1]$, we have:

$$(1-\beta)[W(w) - U] = \beta[J(w,k) - V(k)]. \tag{10}$$

Because renegotiation over the wage is not allowed after the injury or illness shock, the wage is determined, taking account of the possibility of a worker being injured in the future. Substituting equations (3) and (7) into equation (10) gives:

$$w = \beta\left[\frac{r+\alpha+\delta}{r+\alpha+\delta+\lambda(k)}p - (r+\delta)k\right] + (1-\beta)rU.$$

$\frac{r+\alpha+\delta}{r+\alpha+\delta+\lambda(k)}p$ is the expected productivity, and $(r+\delta)k$ is the cost of capital for workplace safety, accounting for the future discount rate r and the rate of destroying δ. According to the Nash bargaining rule, the wage is determined by the weighted-average of the expected profit (the first parentheses on the right-hand side) and the reservation wage rU. From equations (4), (9) and (10), the reservation wage rU can be expressed as $rU = \frac{\beta}{1-\beta}(r+\delta)k\theta$. Substituting this into the above wage equation yields:

$$w = \beta\left[\frac{r+\alpha+\delta}{r+\alpha+\delta+\lambda(k)}p - (1-\theta)(r+\delta)k\right]. \tag{11}$$

There are three effects of capital for workplace safety k on the wage. An increase in k raises the expected productivity $\frac{r+\alpha+\delta}{r+\alpha+\delta+\lambda(k)}p$ because of the lower likelihood that the employed worker is injured or taken ill at the work site and absent from work.[8] $\theta(r+\delta)k$ is

[8]Suppose that renegotiation is allowed after the injury or illness shock. The wage for an active worker is computed by $w = \beta\left[p - (1-\theta)(r+\delta)k\right]$ while the wage for an absent worker because injury or illness is $w = -\beta(1-\theta)(r+\delta)k$. As we would expect, the wage is higher when the renegotiation is allowed than when it is not. The wage for an absent worker turns to be negative because firms have to recover the cost for *ex ante* investment in capital for workplace safety from the absent worker. Unless the expected capital gain of the absent worker by recovering from injury or illness to the work site is sufficiently large,

the total cost of capital for workplace safety per unemployed worker incurred by the firm. An increase in k provides the worker with extra bargaining power, directly reflecting the higher wage. These two effects therefore lead to an increase in the wage. On the other hand, an increase in k raises the cost of capital $(r+\delta)k$ and thereby lowers the expected profit, leading to a decrease in the wage. As seen in the wage equation (11), if $\theta \geq 1$; that is, the measure of vacancies is larger than the number of unemployed workers, the positive effects are unambiguously dominant over the negative effect.

Substituting this wage equation (11) into (7) gives the value for an occupied job of being active:

$$J(k,\theta) = \frac{(1-\beta)(r+\alpha+\delta)p}{(r+\delta)(r+\alpha+\delta+\lambda(k))} + \frac{\beta(1-\theta)(r+\delta)k}{r+\delta}.$$

The first term on the right-hand side shows that an increase in capital for workplace safety k raises the expected productivity and thereby the value for an occupied job of being active. The second term shows that an increase in k lowers the value of an occupied job if $\theta \geq 1$ because the higher wage is burdensome for firms. If $\theta < 1$, an increase in k strictly raises the value of an occupied job. The free entry condition can be rewritten by substituting this into equation (9):

$$\frac{q(\theta)}{r+\delta+q(\theta)}\left[\frac{(1-\beta)(r+\alpha+\delta)p}{(r+\delta)(r+\alpha+\delta+\lambda(k))} + \frac{\beta(1-\theta)(r+\delta)k}{r+\delta}\right] = k. \tag{12}$$

We move to the problem regarding the optimal choice of capital for workplace safety. A firm chooses the optimal capital for workplace safety per job vacancy to maximize the expected value of a vacancy:

$$\max_k V(k) - k \Longrightarrow \max_k \frac{q(\theta)}{r+\delta+q(\theta)} J(k,\theta) - k.$$

The first-order condition yields:

$$\frac{q(\theta)}{r+\delta+q(\theta)}\left[-\frac{(1-\beta)(r+\alpha+\delta)\lambda'(k)p}{(r+\delta)(r+\alpha+\delta+\lambda(k))^2} + \beta(1-\theta)\right] = 1. \tag{13}$$

it is better off for the absent worker to quit and become unemployed.

The left-hand side term represents the expected marginal value of an occupied job with respect to the capital for workplace safety while the right-hand side term is its marginal cost, that is, the price of capital normalized to one.

According to the second-order condition, the slope of the marginal value of an occupied job described above is computed as:

$$\frac{\partial^2 V(k)}{\partial k^2} = \left[\frac{q(\theta)}{r+\delta+q(\theta)}\right]\frac{\partial^2 J(k,\theta)}{\partial k^2} = \left[\frac{q(\theta)}{r+\delta+q(\theta)}\right]\left[-\frac{(1-\beta)(r+\alpha+\delta)p}{r+\delta}\right] \times\frac{\lambda''(k)(r+\alpha+\delta+\lambda(k))-2(\lambda'(k))^2}{(r+\alpha+\delta+\lambda(k))^3}.$$

If $\lambda''(k)(r+\alpha+\delta+\lambda(k))-2(\lambda'(k))^2>0$ for all k, then this is sufficient to show that the optimal capital level maximizes the value of a vacancy.[9]

Next, steady-state conditions are illustrated to derive the unemployment rate and the rate of absent employed workers. Let u and a denote fractions of unemployed workers and employed workers who are absent from work because of work-related injuries or illnesses, respectively. The steady-state conditions require first of all that the inflow rate to the unemployment pool equals the outflow rate from it, and secondly that the inflow rate to the absent pool equals the outflow rate from it,

$$\theta q(\theta)u = \delta(1-u-a)+\delta a = \delta(1-u),$$

$$\text{and}\quad (\alpha+\delta)a = \lambda(k)(1-u-a).$$

Then we obtain:

$$\begin{aligned} u &= \frac{\delta}{\delta+\theta q(\theta)}, \\ \text{and } a &= \left(\frac{\lambda(k)}{\alpha+\delta+\lambda(k)}\right)\left(\frac{\theta q(\theta)}{\delta+\theta q(\theta)}\right). \end{aligned} \tag{14}$$

[9] If $\lambda(k)=1/(1+k)$, for example, $\lambda''(k)(r+\alpha+\delta+\lambda(k))-2(\lambda'(k))^2 = 2(r+\alpha+\delta)/(1+k)^3>0$

The fraction of employed workers who are actively engaged in work is therefore computed by:

$$1 - u - a = \left(\frac{\alpha + \delta}{\alpha + \delta + \lambda(k)} \right) \left(\frac{\theta q(\theta)}{\delta + \theta q(\theta)} \right). \tag{15}$$

As one would expect, the higher labor market tightness θ lowers the unemployment rate, u but raises the fraction of absent employed workers a because the number of employed workers is larger. An increase in capital for workplace safety k lowers the fraction of absent employed a because workplace safety conditions are improved, but k does not affect u. A change of k just influences the distribution between active and absent employed workers.

The nature of equilibrium is characterized by the free entry condition, the first-order condition to determine the optimal level of capital, and the steady-state conditions. Equations (12), (13) and (14) provide a complete description of equilibrium to solve for the vector (k, θ, u, a). For convenience, these equilibrium conditions are summarized below.

(i) First-order condition (equation (12))

$$\frac{q(\theta)}{r + \delta + q(\theta)} \left[-\frac{(1-\beta)(r + \alpha + \delta)\lambda'(k)p}{(r+\delta)(r + \alpha + \delta + \lambda(k))^2} + \beta(1-\theta) \right] = 1,$$

(ii) Free entry condition (equation (13))

$$\frac{q(\theta)}{r + \delta + q(\theta)} \left[\frac{(1-\beta)(r + \alpha + \delta)p}{(r+\delta)(r + \alpha + \delta + \lambda(k))} + \beta(1-\theta)k \right] = k,$$

and (iii) Steady-state conditions (equation (14))

$$\begin{aligned} u &= \frac{\delta}{\delta + \theta q(\theta)}, \\ \text{and } a &= \left(\frac{\lambda(k)}{\alpha + \delta + \lambda(k)} \right) \left(\frac{\theta q(\theta)}{\delta + \theta q(\theta)} \right). \end{aligned}$$

We investigate the characterizations of the equilibrium by examining the comparative statics in the next section.

3 The Relationship between Unemployment and the Incidence of Injury or Illness

Our concerns focus on a relationship between unemployment rate and the incidence of work-related injury or illness via exogenous parameter changes. We first pick up productivity p as a key parameter. This exercise helps provide an explanation about the puzzling results obtained from data findings.

Does productivity improvement encourage firms to raise the amount of capital for workplace safety as well as to enter the labor market? The main purpose in this subsection is to illustrate changes in workplace safety and labor market conditions in response to productivity improvement. Using (i) first-order conditions and (ii) free entry conditions, the comparative statics system is described. The appendix section shows the analytical details.

Proposition 1 *The comparative statics analysis provides the following characterizations:*

$$\frac{dk}{dp} \lesseqgtr 0, \text{ and } \frac{d\theta}{dp} \geqslant 0.$$

As one would expect, an increase in productivity p raises labor market tightness θ. More firms enter the labor market and create vacancies because the productivity improvement leads to an increase in profit, thus resulting in an increase in θ. If $\beta = 1$; that is, there is no bargaining power over the wage determination for firms, firms do not have incentives to enter the labor market despite the fact that productivity improves, and therefore $d\theta/dp = 0$.

An increase in productivity p has two effects on the optimal capital for workplace safety k. On the one hand, because more vacancies created newly due to productivity improvement make it more difficult for firms to find unemployed workers, firms that have to buy capital for workplace safety k *before* matching are discouraged from doing so.[10] On the other hand, an increase in productivity implies an increase in profit that

[10] In a case that firms buy capital for workplace safety *after* hiring (*ex post* investment in safety), this negative effect of productivity on capital for workplace safety disappear.

firms would have earned without workers' absence from work. It therefore encourages firms to increase their capital for workplace safety k to prevent employed workers from being injured or taken ill at the work sites. Which effect is dominant is theoretically ambiguous. According to the comparative statics analysis shown in Appendix, we find that the positive effect on capital is more likely to dominate as the injury rate $\lambda(k)$ is more inelastic; that is, as an marginal decrease in $\lambda(k)$ is larger, firms are encouraged to buy more capital for workplace safety k to reduce the likelihood that accidents occur. If $\beta = 1$, both opposite effects are canceled out, and therefore, productivity improvement does not affect the amount of capital for workplace safety ($dk/dp = 0$).

We next consider the effect on the unemployment rate and the fraction of workers who are absent from work because of work-related injuries or illnesses. Using (iii) steady-state conditions (equation (14)), the total derivatives of these equations are taken with respect to p:

$$\frac{du}{dp} = \underbrace{\left\{-\frac{\delta}{[\delta + \theta q(\theta)]^2}\right\}\left(\frac{\partial \theta q(\theta)}{\partial \theta}\right)}_{-}\underbrace{\left(\frac{d\theta}{dp}\right)}_{+} \leqslant 0, \tag{16}$$

and

$$\begin{aligned} \frac{da}{dp} \quad = \quad & \underbrace{\left(\frac{(\alpha+\delta)\lambda'(k)}{[\alpha+\delta+\lambda(k)]^2}\right)\left(\frac{\theta q(\theta)}{\delta + \theta q(\theta)}\right)}_{-}\underbrace{\left(\frac{dk}{dp}\right)}_{+/-} \\ & + \underbrace{\left(\frac{\lambda(k)}{\alpha+\delta+\lambda(k)}\right)\left(\frac{\delta}{[\delta+\theta q(\theta)]^2}\right)\left(\frac{\partial \theta q(\theta)}{\partial \theta}\right)}_{+}\underbrace{\left(\frac{d\theta}{dp}\right)}_{+}. \end{aligned} \tag{17}$$

The term on the right-hand side in equation (16) and the second term on the right-hand side in equation (17) show that higher labor market tightness caused by productivity improvement lowers the unemployment rate u but raises the fraction of absent employed workers a. The entry of more firms raises the likelihood for unemployed workers of obtaining a job and thereby the employment rate. An increase in the employment rate results directly in an increase in the fraction of absent employed workers because of work-related injuries or illnesses, holding the transition rate of work-related injury or illness fixed. This is referred to as the *scale effect*.

Looking at the first term on the right-hand side in equation (17), in a case that the productivity improvement raises the amount of capital for workplace safety k, the fraction of absent employed workers is lower. This is because an increase in k reduces the likelihood that employed workers are injured or taken ill at the work sites and absent from work $\lambda(k)$. In the opposite case, productivity improvement leads to the higher fraction of absent employed workers through a decrease in the amount of capital for workplace safety. This effect through capital for workplace safety is referred to as the *capital effect*. It causes a shift to active employed workers $(1 - u - a)$ from absent employed workers a in the distribution of employees, holding the transition rates between employment and unemployment fixed. As seen in equation (16), there is no capital effect on u. Its effect affects only the distribution of employees between active workers and absent employed workers.

The implications are summarized below:

Proposition 2 *As productivity improves,(a) (scale effect) the unemployment rate is lower, but the fractions of absent and active employed workers are higher; (b) (capital effect 1) if firms are encouraged to purchase more (less) capital for workplace safety, the fraction of absent employed workers is lower (higher), but the fraction of active employed workers is higher (lower); and (c) (capital effect 2) there is no effect on the unemployment rate through a change of capital for workplace safety.*

Although the scale effect of productivity improvement on u is unambiguously negative, the combined capital and scale effects on a cannot be determined with certainty if $dk/dp > 0$ from equation (17). If the capital effect is dominant over the scale effect, the overall effect of productivity is negative on a. Therefore, u and a are positively correlated through productivity improvement. On the other hand, if the scale effect is dominant over the capital effect, u and a are negatively correlated. What are the factors determining the magnitude relation between the capital and scale effects? One of them is the elasticity of $\lambda(k)$; that is, as $\lambda(k)$ is inelastic with respect to k, it is more likely that the capital effect dominates the the scale effect, thus resulting in the positive relationship between u and a. These implications are summarized.

Proposition 3 *Suppose $dk/dp > 0$. As productivity improves, (a) if the capital effect is dominant over the scale effect, the unemployment rate, u and the fraction of workers who are absent from work because of work-related injuries or illnesses, a move downward in the same direction; and (b) in the reverse case, the unemployment rate, u moves downward while the fraction of workers who are absent from work because of work-related injuries, a moves upward.*

It has been argued by policy makers and OSH specialists that the flow of employed who become injured or ill at the work sites is positively associated with the employment size according to the scale effect. This view was supported by Robinson (1988) and Poteet and Didonato (2001). In other words, it implies that the flow rate of employed workers who become injured or ill and the unemployment rate are *negatively* correlated, holding the labor force fixed.

However, Ussif (2004) showed an opposite view that despite a steady increase in the number of employed workers, the number of work-related injuries has declined from 1970 to 1999 using time-series data from selected countries (Canada, Finland, France, US, and Sweden). That is, he implied a *positive* relationship between the unemployment rate and the flow rate of employed workers who become injured.

What are those arguments omitting to be unable to explain this relationship between the unemployment rate and the flow rate of absent employed found in the data? Our model incorporating the capital effect through a determinant of capital for workplace safety k in the search-matching framework provides some implications consistent with the data. The key parameter is productivity p, and capital and scale effects on u and the flow rate of absent employed workers $\lambda(k)(1-u-\alpha)$ alike illustrate a different relationship between them.

According to equation (15), the flow rate of absent employed workers because of work-related injuries or illnesses, y can be expressed as,

$$y = \lambda(k)(1-u-a) = \lambda(k)\left(\frac{\alpha+\delta}{\alpha+\delta+\lambda(k)}\right)\left(\frac{\theta q(\theta)}{\delta+\theta q(\theta)}\right).$$

Then, the total derivative of this flow equation are taken with respect to p:

$$\begin{aligned}\frac{dy}{dp} &= \underbrace{\lambda'(k)\left(\frac{\alpha+\delta}{\alpha+\delta+\lambda(k)}\right)^2\left(\frac{\theta q(\theta)}{\delta+\theta q(\theta)}\right)}_{-}\underbrace{\left(\frac{dk}{dp}\right)}_{+/-} \\ &\quad \underbrace{+\lambda(k)\left(\frac{\alpha+\delta}{\alpha+\delta+\lambda(k)}\right)\left(\frac{\delta}{[\delta+\theta q(\theta)]^2}\right)\left(\frac{\partial\theta q(\theta)}{\partial\theta}\right)}_{+}\underbrace{\left(\frac{d\theta}{dp}\right)}_{+}.\end{aligned} \tag{18}$$

The first term on the right-hand side of equation (18) shows the capital effect; that is, in a case that firms buy more capital for workplace safety k in response to productivity improvement, the probability of being injured or taken ill at the work site $\lambda(k)$ is lower, resulting in a decrease in the flow rate of absent employed workers because of work-related injuries or illnesses to the absent pool. The same argument can be applied in the reverse case. The second term reveals the scale effect; that is, as more firms enter the market and create vacancies due to productivity improvement, the flow rate of absent employed workers increases because of the increasing number of employed workers.

In a case of $dk/dp > 0$, if the scale effect is dominant over the capital effect, the productivity improvement leads to a decrease in the unemployment rate u but an increase in the flow rate of absent employed workers $\lambda(k)(1-u-a)$. These two rates are negatively correlated, as argued by Robinson (1988) and Poteet and Didonato (2001). If productivity is pro-cyclical, this implication is consistent with the empirical finding from Arai and Skogman Thoursie (2005) showing that the extent of sickness absence is strongly pro-cyclical. In the reverse case, throughout a change in capital for workplace safety k in response to a productivity shock u and $\lambda(k)(1-u-a)$ alike move downward in the same direction. This implication is consistent with the data finding from Ussif (2004). The sign of y depends entirely on which is more dominant, the capital effect or the capital effect. The next proposition summarizes:

Proposition 4 *Suppose $dk/dp > 0$. As productivity improves, (a) if the capital effect is dominant over the scale effect, the unemployment rate, u and the flow rate of workers who are absent from work because of work-related injuries or illnesses, $\lambda(k)(1-u-a)$ move downward in the same direction; and (b) in the reverse case, the unemployment rate, u moves downward while the flow rate of workers who are absent from work because of work-related injuries or illnesses, $\lambda(k)(1-u-a)$ moves upward.*

In Section 5, the quantitative analysis is provided.

4 Applications

This section presents the effects of various policy parameters: replacement rate, disability insurance payment, disability insurance premium, and unemployment benefits. These impacts illuminate firms' incentives for determinants of job creation and the amount of capital for workplace safety in response to policy changes. Here is with the emphasis on the impact of unemployment benefits on workplace safety. It appears that the linkage between unemployment benefits and workplace safety has not at least thus far been argued. This comparative statics exercise contributes to illuminating its linkage.

4.1 The Extended Model

An employed worker who is actively engaged in work instantaneously earns w as heretofore, but an employed worker who is absent from work because of work-related injury or illness is recompensed not fully but partially for her/his loss by ρw where ρ represents the replacement rate ($\rho \in (0,1)$). An unemployed worker now receives the unemployment insurance benefits z. The value functions for a worker are modified by:

$$rW(w) = w + \lambda(k)[W_a(w) - W(w)] + \delta[U - W(w)],$$

$$rW_a(w) = \rho w + \alpha[W(w) - W_a(w)] + \delta[U - W_a(w)],$$

and

$$rU = z + \theta q(\theta)[W(w) - U]. \tag{19}$$

We assume that the compensation for work-related injury or illness ρw exceeds the unemployment insurance benefits z, which is more realistic. Since workers are risk-neutral, unemployment benefits are considered just as a subsidy. In a similar manner, the value functions for a firm are:

$$rJ(w,k) = p - (1+t)w + \lambda(k)[J_a(w,k) - J(w,k)] - \delta J(w,k),$$

$$rJ_a(w,k) = -\rho w + \phi w - tw + \alpha[J(w,k) - J_a(w,k)] - \delta J_a(w,k),$$

and

$$rV(k) = q(\theta)[J(w,k) - V(k)] - \delta V(k). \tag{20}$$

Firms that hire workers take out disability insurance and pay the insurance premium tw. A firm has to recompense an injured worker for her/his loss by part of the wage ρw but receives ϕw in disability insurance benefits. We assume $1 > \rho > \phi > t > 0$; that is, the disability insurance program does not fully cover a firm's loss, and the insurance premium is lower than the insurance payment.

Using equations (19) - (20) and the free entry condition ($V(k) = k$), the wage is solved according to the Nash bargaining rule:

$$w = \frac{\beta[(r+\alpha+\delta]p - (1-\theta)(r+\delta)(r+\alpha+\delta+\lambda(k))k] + (1-\beta)[r+\alpha+\delta+\lambda(k)]z}{\beta[(1+t)(r+\alpha+\delta) + \lambda(k)(t+\rho-\phi)] + (1-\beta)[r+\alpha+\delta+\rho\lambda(k)]}.$$

Be aware that this wage equation is reduced to equation (11) in a case of $\rho = 1$ and $\phi = t = z = 0$. Increases in the replacement rate ρ and the disability insurance premium rate t encourage firms to exit the labor market, thus lowering labor demand and thereby the wage. In contrast, an increase in the disability insurance payment rate ϕ raises labor demand by encouraging firms' entry, leading to an increase in the wage. For simplicity, we consider the take-it-or-leave-it wage ($\beta = 0$) from now on[11]:

$$w = \frac{[r + \alpha + \delta + \lambda(k)]z}{r + \alpha + \delta + \rho\lambda(k)}. \tag{21}$$

This is the same as the reservation wage. An increase in capital for workplace safety, k lowers the wage. An increase in k lowers the likelihood that an employee is injured or taken ill at the work site and raises the expected value of being employed relative to the value of being unemployed. The state of employment is more attractive, and firms are allowed to capture the whole surplus produced by matched pairs, thus resulting in decreases in the reservation wage and thereby the wage[12].

From equations (20) and equation (21), we obtain the value of a job being occupied and active:

$$J(k) = \frac{(r + \alpha + \delta)p}{(r + \delta)[r + \alpha + \delta + \lambda(k)]} - \frac{[(1 + t)(r + \alpha + \delta) + \lambda(k)(t + \rho - \phi)]z}{(r + \delta)[r + \alpha + \delta + \rho\lambda(k)]}. \tag{22}$$

Similarly to Section 2, the nature of equilibrium is characterized by (i) the first-order condition ($V'(k) = 1$), (ii) the free entry condition ($V(k) = k$) and (iii) the steady-state conditions:

(i) the first-order condition

$$\frac{q(\theta)J'(k)}{r + \delta + q(\theta)} - 1 = 0,$$

(ii) the free entry condition

[11] The holdup problem does not arise because there is no bargaining power over the wage determinantion for workers.

[12] If $\rho = 1$ as in the basic model described in Section 2, the wage is reduced to z and does not depend on the amount of capital for workplace safety, k. Because the value for a worker being actively employed and the value of being absent from work due to work-related injury or illness are the same in this case, the likelihood of being injured or taken ill at the work site, which depends on k, does not affect the reservation wage.

$$\frac{q(\theta)J(k)}{r+\delta+q(\theta)} - k = 0,$$

and (iii) Steady-state conditions

$$\begin{aligned} u &= \frac{\delta}{\delta+\theta q(\theta)}, \\ a &= \left(\frac{\lambda(k)}{\alpha+\delta+\lambda(k)}\right)\left(\frac{\theta q(\theta)}{\delta+\theta q(\theta)}\right). \end{aligned}$$

The first-order and the second-order conditions are presented in the appendix section. The steady-state conditions are the same as those in Section 2. The comparative statics studies show the characterizations of the equilibrium in response to the changes of policy parameters.

4.2 Comparative Statics Effects

We next explore the effects of policy parameters (replacement rate, disability insurance payment and premium, and unemployment benefit) on capital for workplace safety and labor market tightness.

According to the comparative statics analysis, we obtain the following results:

Proposition 5 *(a) replacement rate*

$$\frac{dk}{d\rho} \lesseqgtr 0 \quad and \quad \frac{d\theta}{d\rho} \lesseqgtr 0,$$

(b) disability insurance payment rate

$$\frac{dk}{d\phi} < 0 \quad and \quad \frac{d\theta}{d\phi} > 0,$$

(c) disability insurance premium rate

$$\frac{dk}{dt} > 0 \quad and \quad \frac{d\theta}{dt} < 0,$$

and (d) unemployment benefit

$$\frac{dk}{dz} \lesseqgtr 0 \quad \textit{and} \quad \frac{d\theta}{dz} < 0.$$

The appendix section shows analytical details.

(a) replacement rate

There are two effects of ρ on k and θ to be considered: direct effects and indirect effect via w. We begin the direct effects. As the replacement rate ρ is higher, firms have to bear the more burden of the recompense for losses from work-related injuries and illnesses ρw, which encourages firms to exit the labor market, which lowers θ. Less competitiveness induces firms to buy more capital for workplace safety because it is more likely for firms to meet unemployed workers. At the same time, it implies an increase in the cost that firms would have incurred if accidents had occurred to employed workers at the work sites. Firms thus have more incentives to increase the amount of k to reduce the likelihood of being injured or taken ill at the work site.

The indirect effects show the opposite story. An increase in ρ lowers the wage according to equation (21). As absent employed workers are more sufficiently recompensed for their losses, the state of employment is more attractive for workers. In a case of which workers are allowed to have power over wage bargaining, unemployed workers then accept the lower wage. Firms bear the less burden of the recompense for losses from work-related injuries and illnesses ρw, encouraging firms to enter and thereby raising θ. More severe competitiveness discourages firms from buying capital for workplace safety. A decrease in the wage lowers the cost that firms would have incurred if accidents had occurred, which also discourages firms from buying capital for workplace safety. In addition, an decrease in the wage raises the profit, and therefore firms are induced to raise capital for workplace safety because of an increase in the marginal return to capital for workplace safety. Putting differently, an increase in the profit encourages firms' entry, and an increase in the number of rivals lowers the amount of capita for workplace safety.

(b) Disability Insurance Payment Rate

An increase in the disability insurance payment rate ϕ reduces the burden of the recompense for losses from work-related injuries and illnesses, which therefore induces firms to lower the amount of k and encourages firms' entry to the market. An increase in job vacancies due to firms' entry discourages firms from purchasing capital for workplace safety.

(c) Disability Insurance Premium Rate

An increase in the disability insurance premium rate, t implies an increase in the cost that firms would have incurred if accidents had occurred, which thereby induces firms to increase the amount of capital for workplace safety to prevent accidents at the work sites from the value function of an absent worker. Putting differently, an increase in t lowers the profit, which instead induces firms to lower the amount of capital for workplace safety because of a decrease in its marginal return. According to the comparative static analysis shown in Appendix, the former effect is dominant over the latter effect.

Firms are encouraged to exit the labor market by an increase in the recompense for workplace injury and illness and a decrease in the profit. Less competitiveness allows firms to provide more chances to meet unemployed workers. *Ex ante* investment in workplace safety is less likely to be idle, therefore providing firms with the incentive to purchase more capital for workplace safety k.

(d) Unemployment Benefit

As seen in standard matching models, an increase in unemployment benefits z lowers labor market tightness θ. The intuition behind this result is that an increase in z raises the reservation wage of workers and thereby the wage, which encourages firms to exit.

The linkage between unemployment benefits and workplace safety is considered here. The comparative statics study shows that capital for workplace safety k is positively or negatively associated with unemployment benefits z. According to equation (21), an increase in z raises the wage. Because the state of unemployment is more attractive for workers, unemployed workers then raises their reservation wage. It implies an increase in the cost that firms would have incurred if their employed workers had been injured or taken ill at the work sites and been absent from work because the compensation is positively proportional to the wage (ρw). Firms have more incentives to increase capital for workplace safety to reduce the likelihood of being injury or taken ill at the work site. However, an increase in the wage implies a decrease in the profit, lowering the expected gain of investing in workplace safety. Therefore, it leads to a decrease in the amount of capital for workplace safety. Putting oppositely, a decrease in the profit discourages firms from entering the labor market, and less competitiveness provides firms with incentives to purchase capital for workplace safety. If the positive effects on k are dominant over the negative effect, the novel result is that the unemployment benefits have a positive effect on capital for workplace safety, implying a decrease in the likelihood of being injured or taken ill at the work site. The unemployment benefits encourage firms to improve working conditions.

(e) Capital and Scale Effects

How do these policy parameters affect the unemployment rate and the fraction of workers who are absent from work because of work-related injuries or illnesses? The total derivatives of equation (14) are taken with respect to $x = \{\rho, \phi, t, z\}$:

$$\frac{du}{dx} = \underbrace{\left\{-\frac{\delta}{[\delta + \theta q(\theta)]^2}\right\}\left(\frac{\partial \theta q(\theta)}{\partial \theta}\right)}_{-}\left(\frac{d\theta}{dx}\right),$$

and

$$\begin{aligned} \frac{da}{dx} &= \underbrace{\left(\frac{(\alpha+\delta)\lambda'(k)}{[\alpha+\delta+\lambda(k)]^2}\right)\left(\frac{\theta q(\theta)}{\delta+\theta q(\theta)}\right)}_{-}\left(\frac{dk}{dx}\right) \\ &\quad + \underbrace{\left(\frac{\lambda(k)}{\alpha+\delta+\lambda(k)}\right)\left(\frac{\delta}{[\delta+\theta q(\theta)]^2}\right)\left(\frac{\partial \theta q(\theta)}{\partial \theta}\right)}_{+}\left(\frac{d\theta}{dx}\right), \\ x &= \{\rho, \phi, t, z\}. \end{aligned}$$

Comparative statics effects are summarized in Table 1. The scale effects of the disability insurance premium rate t and unemployment benefits z are unambiguously positive on the unemployment rate u but negative on the fraction of absent employed workers a. The disability insurance payment rate ϕ has the opposite scale effects on u and a, respectively. The replacement rate ρ has ambiguous scale effects on both u and a.

The capital effect of ϕ on the fraction of absent employed workers a is unambiguously positive through an decrease in capital for workplace safety k. The opposite capital effect of t is then obtained on a. The capital effects of ρ and z are ambiguous on a because it is unclear whether firms raise or lower the amount of capital for workplace safety in response to increases in these parameters.

5 Simple Empirical Evidence

In this section, we attempt to show that, although indirectly, implications of a relationship between unemployment and the number of work-related injured workers are supported by a simple empirical study. Because the amount of capital for workplace safety purchased by firms cannot be observed, there is no way but the capital effect is identified indirectly from a comparison of the effects of labor productivity on unemployment and the number of work-related injured workers.

We show relative variations of unemployment rate, a fraction of employees who have to be absent from work for non-fatal injury to the labor force size, and labor productivity per person with respect to the corresponding trends that are calculated using the moving average. Figures 1-3 display the relative deviations of these variables from their trends, using time-series data from three countries, the US, Germany and Japan. The data from these three countries are obtained from *LABORSTA Internet* (ILO).

Figure 1 shows using the data from the US covering from 1980 to 2001. The unemployment rate and the fraction of work-related injured employees are negatively correlated over the sample period. The second graph compares relative deviations of labor productivity and the unemployment rate. As one would expect, the correlation between these variables is negative over the sample period, which supports a view that productivity improvement raises job opportunities, thereby lowering the unemployment rate. The third graph of Figure 1 makes the same comparison of the relative deviations of productivity and the fraction of work-related injured workers. The correlation between labor productivity and the fraction of work-related injured workers is positive over the sample period; that is, as productivity improved, the fraction of work-related injured workers had risen. These results imply that the scale effect of labor productivity is dominant over the capital effect. Similarly to the US, the same results are, although roughly, obtained using data from Germany covering from 1995 to 2005 (Figure 2); that is, the correlation between the unemployment rate and the fraction of work-related injured employees is negative. We predict that an increase in the number of work-related injured workers is attributable mainly to the larger employment size in the US and Germany.

Figure 3 shows relative deviations of the variables using the Japanese data from 1990 to 2006. We observe different patterns in movements of the deviations between before and after 1995. The correlation between the unemployment rate and the fraction of work-related injured workers after 1995 is similar to those of the US and Germany; that is, these two deviations moved in the opposite direction. On the other hand, it is observed that the deviations moved in the same direction before 1995, implying a positive correlation between the unemployment and the fraction of work-related injured workers. The correlation between labor productivity and the fraction of work-related injured workers turns out to be opposite on reaching 1995; that is, as labor productivity improved, the fraction of work-related injured workers increased as well after 1996, but the opposite pattern is seen before 1995. This implies that similar to the US and Germany, the scale effect is dominant after 1996, but that before 1995, the capital effect is dominant over the scale effect. We predict that increasing productivity encouraged firms to purchase more capital for workplace safety to reduce the number of work-related accidents. It might be interrupted that the fraction of work-related injured workers would rather be determined by the amount of capital for workplace safety than by the employment size.

We estimate equations of increased rates of work-related injured workers and the unemployment rate to reinforce results obtained from data analysis. The purpose of this empirical work is to, although indirectly, identify the capital effect in the sense that productivity improvement encourages firms to purchase capital for workplace safety, leading to a decrease in the number of work-related injured workers.

Table 1 presents empirical results. The first column estimates the equation of the increased rate of work-related injured workers with the OLS method. Since the null hypothesis of no autocorrelation is not significantly rejected, the OLS method is accepted. The second column displays the AR(1) estimate of the increased rate of the unemployment rate. This estimation uses the AR(1) method because that the null hypothesis of no autocorrelation is significantly rejected. The independent variables are the increased rate of labor productivity per person and its interaction term with the year dummy indicating one if the year is before 1995.

Table 1:Estimates of Growth Rates of Work-Related Injured Workers and the Unemployment Rate.

	(1)		(2)	
	OLS ΔInjuries		AR(1) ΔUnemployment	
	Coef.	Std. Err.	Coef.	Std. Err.
ΔProductivity	1.015**	(0.365)	-4.262*	(0.779)
ΔProductivity×year dummy	-2.066*	(0.611)	-0.420	(1.527)
Constant	-0.039*	(0.006)	0.083***	(0.039)
	Value	P-value	Value	P-Value
DW Statistic	2.096		0.895	
Alt. DW Statistic (F value)	0.160	(0.696)	4.801	(0.049)
Breusch-Godfrey (F-value)	0.211	(0.655)	4.572	(0.054)
F value	7.22	(0.008)	16.69	(0.0003)
R^2	0.526		0.720	
Sample	16		16	

* 1%, ** 5%, *** 10% significant. Year dummy is the variable indicating one if the year is before 1995.

The first column shows that as labor productivity improved, the fraction of work-related injured workers declined before 1995 but instead rose after 1995. These results are consistent with observations from Figure 3. According to the second column, regardless of whether the year is before or after 1995, as productivity improved, the unemployment declined, in other words, the employment rate rose. These two estimated results imply two findings. The first one is that after 1995, productivity improvement enlarged the employment size, which also leads to a rise in the fraction of work-related injured workers. Therefore, the scale effect was dominant over the sample period after 1995, and it is predicted that the capital effect was trivial. The second finding is as follows. Before 1995, productivity improvement not only created more job vacancies to enlarge the employment size but also induced firms to purchase more capital for workplace safety to reduce the probability of losing a productive worker due to work-related injury. The former mechanism operates an increase in the fraction of work-related injured workers as seen after 1995, but the latter mechanism leads to a decline in its fraction. We found that the latter effect, in other word, the capital effect is dominant over the former one, that is, the scale effect.

6 Concluding Remarks

This paper allows for firms' decisions on the amount of capital for workplace safety, and trade-off between its cost and the risk of employed workers being absent from work because of work-related injuries or illnesses.

By incorporating firms' decisions on the amount of capital for workplace safety in a search-matching model, we investigated the relationship between unemployment and the incidence of work-related injury or illness. Productivity improvement encourages firms' entry and then raises labor market tightness. It lowers the unemployment rate but instead raises the employment rate, including the fraction of absent employed workers. This scale effect implies a negative relationship between the unemployment rate and the fraction of absent employed workers. However, productivity improvement changes firms' behavior toward a determinant of the amount of capital for workplace safety. Potential firms are induced to enter the market as productivity improves, which lowers the likelihood that firms meet unemployed workers. Firms are thereby discouraged from purchasing capital for workplace safety *ex ante*. On the other hand, it is possible that productivity improvement induces firms to buy more capital for workplace safety because of an increase in the opportunity cost of losing workers due to work-related injuries or illnesses. This capital effect biases the distribution of employed workers toward active employed workers but does not affect the unemployment rate. If the capital effect of productivity is dominant, the relationship between the unemployment rate and the fraction of absent employed workers because of work-related injuries or illnesses is positive. In a similar manner, the sign of the relationship between the unemployment rate and the flow rate of absent employed workers depends on which is more dominant, the scale effect or the capital effect. The determinant of the amount of capital for workplace safety helps provide an explanation about the puzzling results regarding the relationship between the unemployment rate and the number of absent employed workers who are injured or taken ill suggested by Ussif (2004).

We now briefly discuss social efficiency. According to this model setting, firms buy capital for workplace safety *before* meeting workers in their search activity, and therefore it is well-known that *ex ante* investment in workplace safety causes a hold-up problem as mentioned by Acemoglu and Shimer (1999). It is expected that the optimal capital level for workplace safety in the decentralized environment is lower than the one solved by the social planner[13]. It implies that the enforcement of mandates of workplace safety by OSHA is justified to improve social welfare in an environment where a hold-up problem arises.

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Appendices

Comparative Statics Effect (Section 3)

In this appendix, we use comparative statics analysis to explore the effects of productivity p on capital for workplace safety k and labor market tightness θ, using the first-order condition (equation (13)) and the free entry condition (equation (12)). The comparative statics system is given by:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \frac{dk}{dp} \\ \frac{d\theta}{dp} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

where:

$$A_{11} = \left[-\frac{(1-\beta)(r+\alpha+\delta)p}{r+\delta}\right]\left[\frac{\lambda''(k)(r+\alpha+\delta+\lambda(k)) - 2(\lambda'(k))^2}{(r+\alpha+\delta+\lambda(k))^3}\right] \leqslant 0$$

from the second-order condition,

$$A_{12} = -\beta + \frac{(r+\delta)q'(\theta)}{[q(\theta)]^2} < 0,$$

$$A_{21} = -\frac{(1-\beta)(r+\alpha+\delta)\lambda'(k)p}{(r+\delta)(r+\alpha+\delta+\lambda(k))^2} + \beta(1-\theta) - \frac{r+\delta+q(\theta)}{q(\theta)} = 0,$$

from the first-order condition,

$$A_{22} = -\beta k + \frac{(r+\delta)q'(\theta)}{[q(\theta)]^2}k \leqslant 0,$$

$$B_1 = \frac{(1-\beta)(r+\alpha+\delta)\lambda'(k)}{(r+\delta)(r+\alpha+\delta+\lambda(k))^2} \leqslant 0,$$

and

$$B_2 = -\frac{(1-\beta)(r+\alpha+\delta)}{(r+\delta)(r+\alpha+\delta+\lambda(k))} \leqslant 0.$$

The Jacobian determinant is $\nabla_A \equiv A_{11}A_{22} - A_{12}A_{21} > 0$. Then we find:

$$\frac{dk}{dp} = \left(\frac{1}{\nabla_A}\right) \underbrace{\left[-\beta + \frac{(r+\delta)q'(\theta)}{[q(\theta)]^2}\right]}_{-} \underbrace{\left[\frac{(1-\beta)(r+\alpha+\delta)}{(r+\delta)(r+\alpha+\delta+\lambda(k))}\right]}_{+} \underbrace{\left[\frac{\lambda'(k)k+r+\alpha+\delta+\lambda(k)}{r+\alpha+\delta+\lambda(k)}\right]}_{+/-},$$

and

$$\frac{d\theta}{dp} = \frac{A_{11}B_2 - A_{21}B_1}{\nabla_A} \geqslant 0.$$

As the elasticity of $\lambda(k)$ ($\lambda'(k)k/\lambda(k)$) is much lower, it is more likely that $dk/dp > 0$.

Comparative Statics Effect (Section 4)

The first-order condition and the free entry condition are rewritten as:

$$J'(k) - \frac{r+\delta+q(\theta)}{q(\theta)} = 0,$$

$$\text{and } J(k) - \frac{r+\delta+q(\theta)}{q(\theta)}k = 0,$$

where :

$$J(k) = \frac{(r+\alpha+\delta)p}{(r+\delta)[r+\alpha+\delta+\lambda(k)]} - \frac{[(1+t)(r+\alpha+\delta)+\lambda(k)(t+\rho-\phi)]z}{(r+\delta)[r+\alpha+\delta+\rho\lambda(k)]}.$$

Since $J'(k) > 0$, we recognize that the marginal value of vacancy is positive for any k ($V'(k) > 0$). If $\lambda''(k)(r+\alpha+\delta+\lambda(k)) - 2(\lambda'(k))^2 > 0$ for any k, $J''(k) < 0$, which is sufficient to show that the optimal capital level maximizes the value of a vacancy.

The comparative statics analysis is provided below to investigate the effects of the policy parameters:

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} dk \\ d\theta \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \end{bmatrix} \begin{bmatrix} d\rho & d\phi & dt & dz \end{bmatrix},$$

where:

$$C_{11} = J''(k) < 0$$

from the second-order condition,

$$C_{12} = \frac{(r+\delta)q'(\theta)}{[q(\theta)]^2} < 0,$$

$$C_{21} = J'(k) - \frac{r+\delta+q(\theta)}{q(\theta)} = 0$$

from the first-order condition,

$$C_{22} = \frac{(r+\delta)q'(\theta)k}{[q(\theta)]^2} < 0,$$

$$D_{11} = -\left[\frac{\lambda'(k)(r+\alpha+\delta)z}{r+\delta}\right]\left[\frac{t(r+\alpha+\delta+\rho\lambda(k)) + 2\lambda(k)((1-\rho)t-\phi)}{(r+\alpha+\delta+\rho\lambda(k))^3}\right] \lesseqgtr 0,$$

$$D_{21} = \left(\frac{z}{r+\delta}\right)\frac{\lambda(k)[t(r+\alpha+\delta)+\lambda(k)(t-\phi)]}{(r+\alpha+\delta+\rho\lambda(k))^2} \lesseqgtr 0,$$

$$D_{12} = -\frac{\lambda'(k)(r+\alpha+\delta)z}{(r+\delta)(r+\alpha+\delta+\rho\lambda(k))^2} > 0,$$

$$D_{22} = -\frac{\lambda(k)z}{(r+\delta)(r+\alpha+\delta+\rho\lambda(k))} < 0,$$

$$D_{13} = \frac{\lambda'(k)(r+\alpha+\delta)(1-\rho)z}{(r+\delta)(r+\alpha+\delta+\rho\lambda(k))^2} < 0,$$

$$D_{23} = \frac{(r+\alpha+\delta+\lambda(k))z}{(r+\delta)(r+\alpha+\delta+\rho\lambda(k))^2} > 0,$$

$$D_{14} = \frac{\lambda'(k)(r+\alpha+\delta)((1-\rho)t-\phi)}{(r+\delta)(r+\alpha+\delta+\rho\lambda(k))^2} > 0,$$

assuming $\phi \neq (1-\rho)t$,

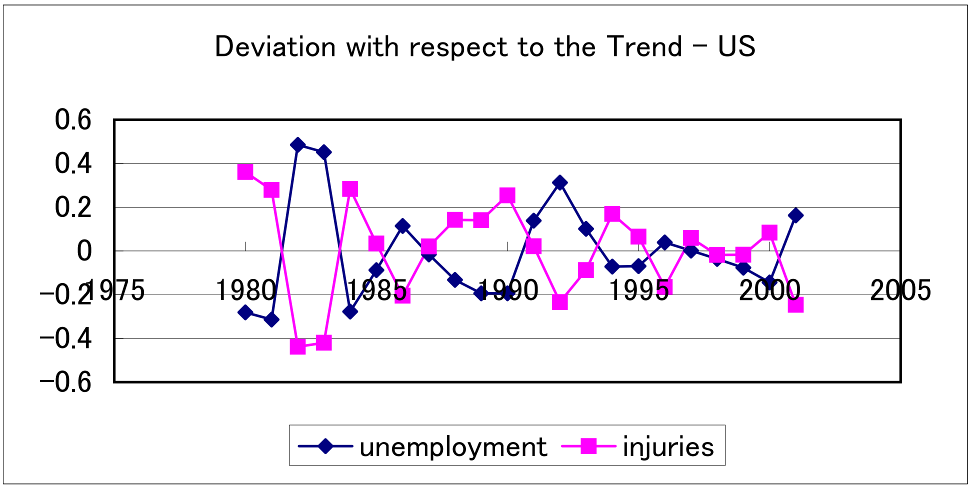
and

$$D_{24} = \frac{(1+t)(r+\alpha+\delta)+\lambda(k)(t+\rho-\phi)}{(r+\delta)(r+\alpha+\delta+\rho\lambda(k))} > 0,$$

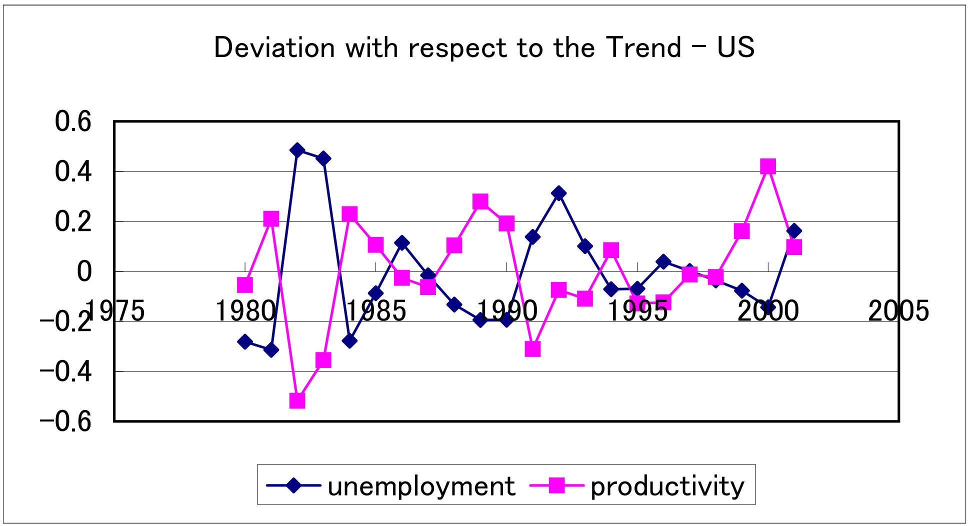
assuming $\phi \neq t+\rho$.

The Jacobian determinant is $\nabla_C \equiv C_{11}C_{22} - C_{12}C_{21} > 0$. We then obtain the implications described in proposition 5.

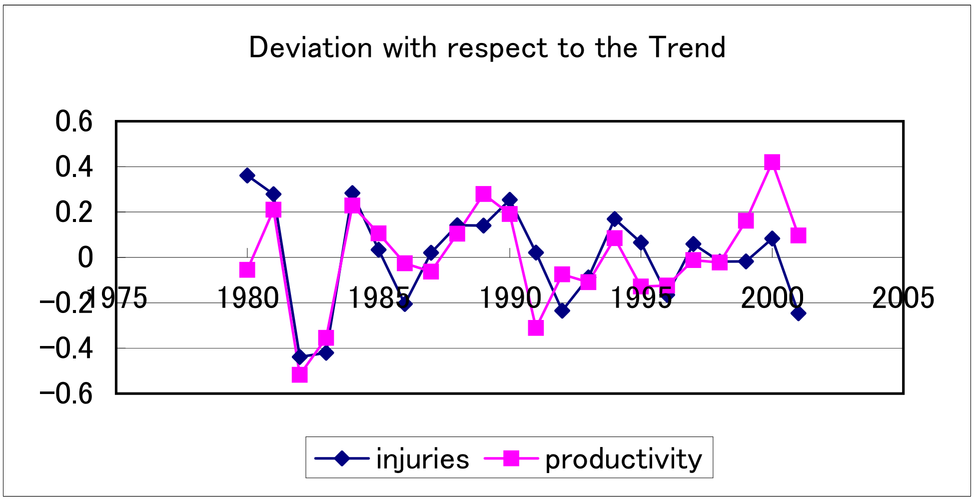
Figure 1



Correlation = -0.944

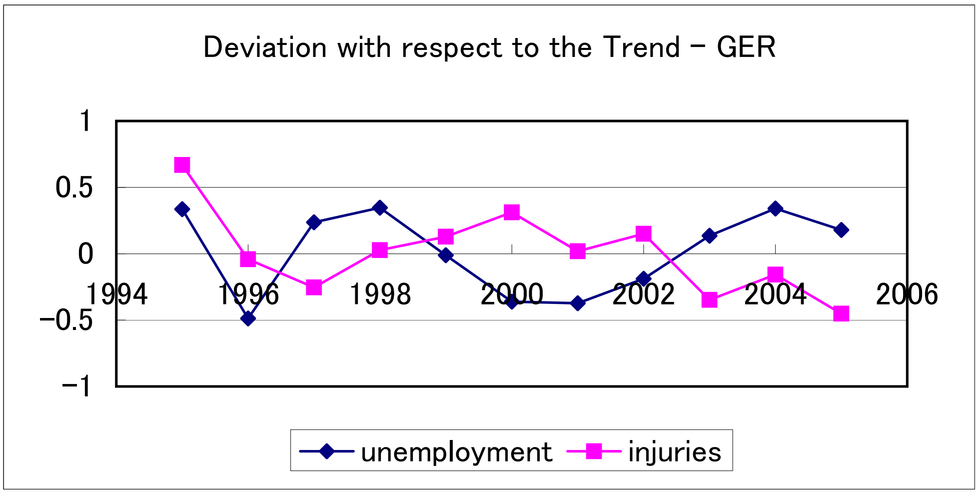


Correlation = -0.777

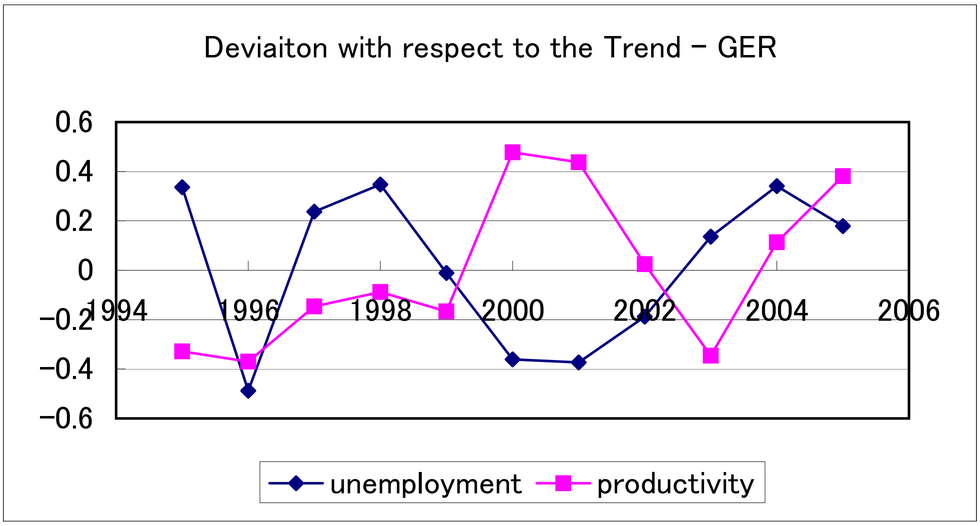


Correlation = 0.644

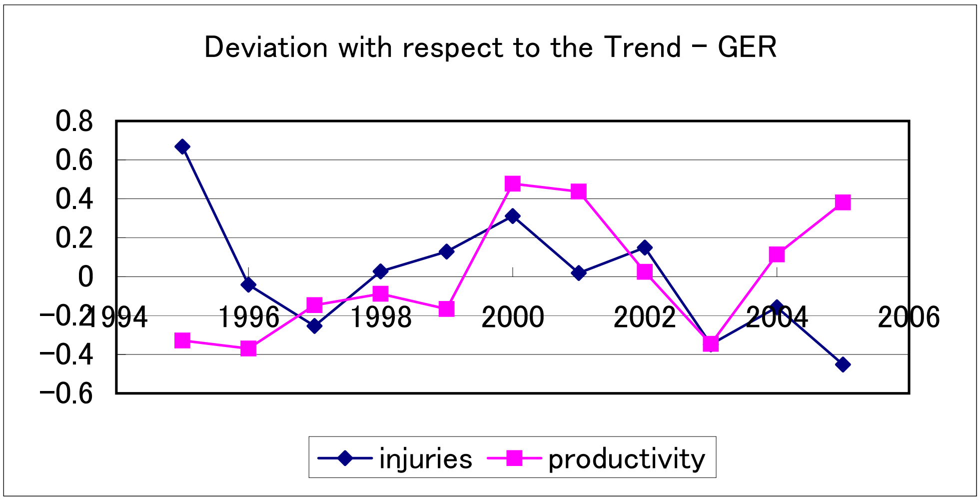
Figure 2



Correlation = -0.138

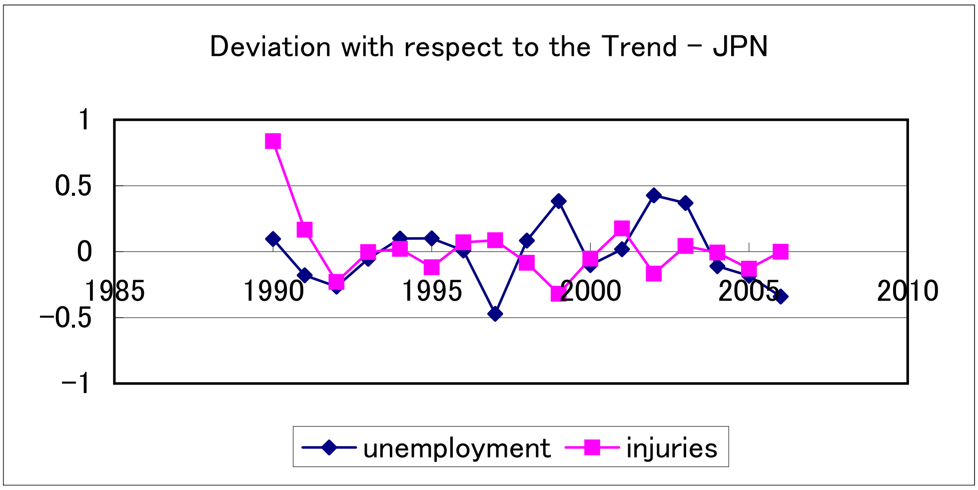


Correlation = -0.274

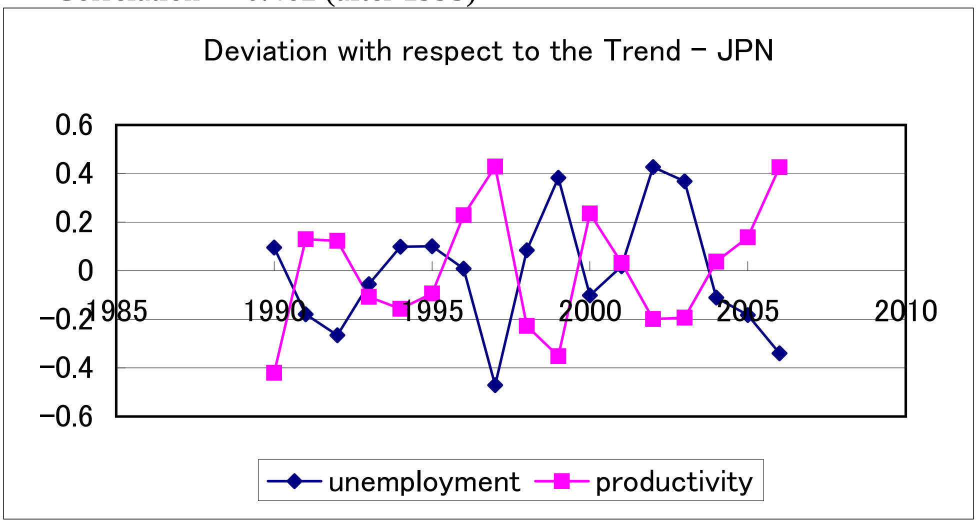


Correlation = -0.105

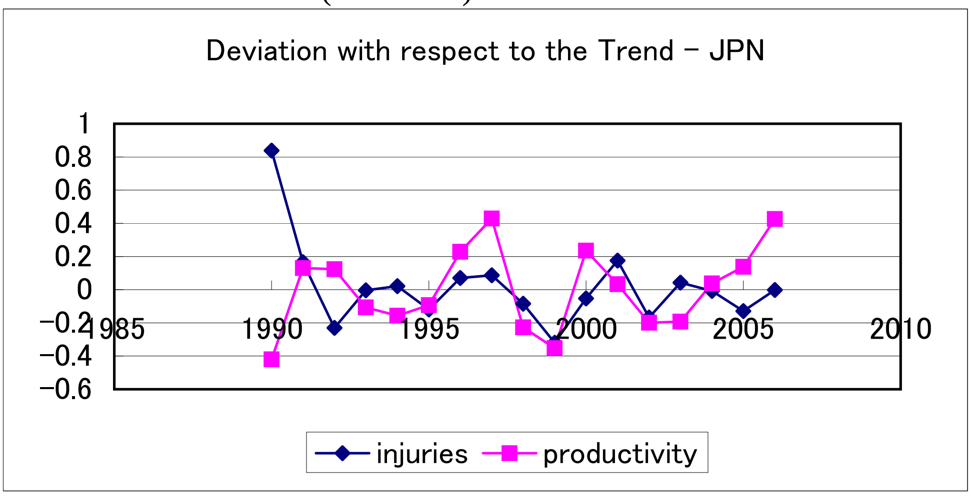
Figure 3



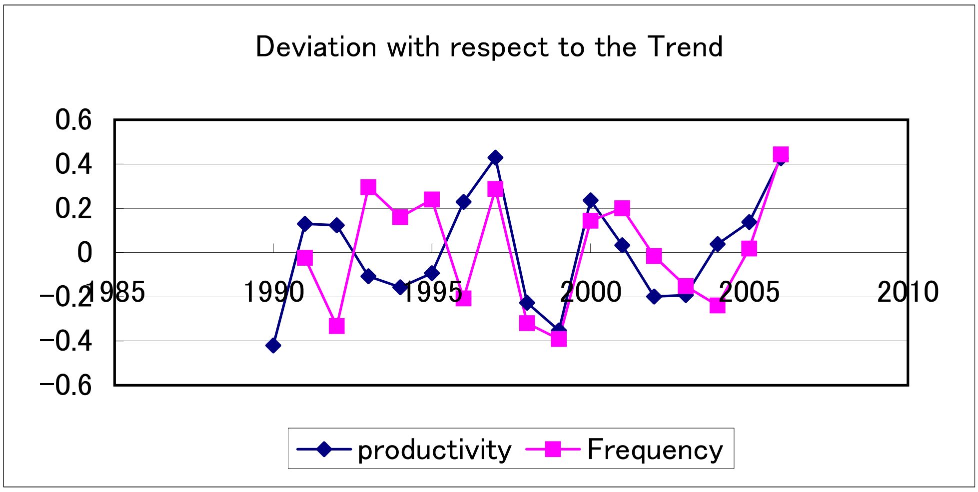
Correlation = -0.089
Correlation = 0.410 (until 1995)
Correlation = -0.462 (after 1995)



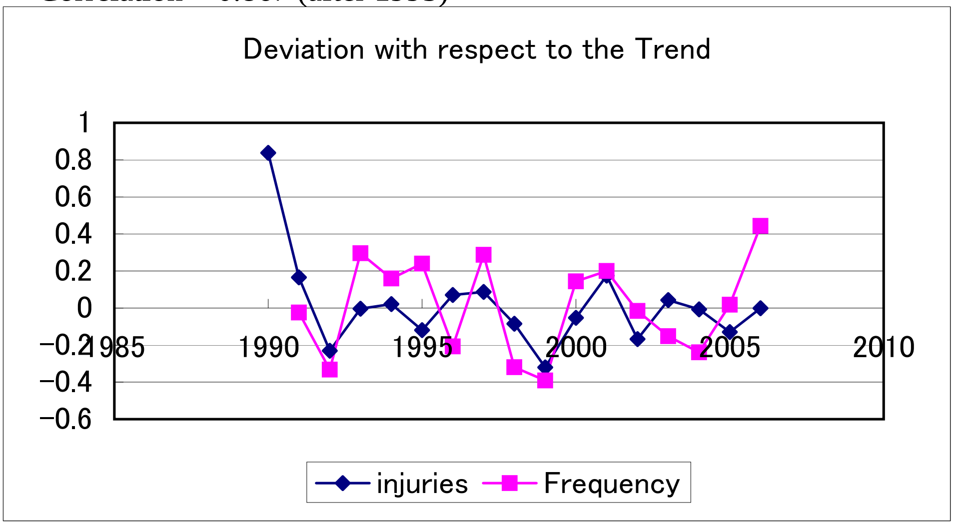
Correlation = -0.823
Correlation = -0.817 (until 1995)
Correlation = -0.926 (after 1995)



Correlation = -0.167
Correlation = -0.754 (until 1995)
Correlation = 0.555 (after 1995)



Correlation = 0.461
Correlation = -0.836 (until 1995)
Correlation = 0.867 (after 1995)



Correlation = 0.445
Correlation = 0.377 (until 1995)
Correlation = 0.574 (after 1995)